Manipulation via Endowments in Exchange Markets with Indivisible Goods*

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Abstract

We consider exchange markets with heterogeneous indivisible goods. We are interested in exchange rules that are efficient and immune to manipulations via endowments (either with respect to hiding or destroying part of the endowment or transferring part of the endowment to another trader). We consider three manipulability axioms: hiding-proofness, destruction-proofness, and transfer-proofness. We prove that no rule satisfying efficiency and hiding-proofness (which together imply individual rationality) exists. For two agents with separable and responsive preferences, we show that efficient, individually rational, and destruction-proof rules exist. However, for some profiles of separable preferences, no rule is efficient, individually rational, and destruction-proof. In the case of transfer-proofness the compatibility with efficiency and individual rationality for the two-agent case extends to the unrestricted domain. If there are more than two agents, for some profiles of separable preferences, no rule is efficient, individually rational, and transfer-proof.

JEL Classification: C71, D63, D71.

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1 Introduction

We consider exchange markets with heterogeneous indivisible objects where each agent is endowed with a set of objects. As an example, one may think of markets where people trade collectibles, for instance stamps, Pockeymon cards, *etc.*. Other applications (see also Pápai, 2003) are exchanges of equipment or tasks among workers or departments of a firm or an organization. A well-known special case of our exchange model are so-called housing markets (Shapley and Scarf, 1974) where each agent is endowed with exactly one object. For housing markets, the so-called top trading rule that assigns the unique core allocation to each housing market satisfies many appealing properties. In particular,

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the top trading rule is *efficient* and *strategy-proof* [no agent ever benefits from misrepresenting her preferences] (Roth, 1982). Moreover, it is the only rule satisfying *efficiency*, *strategy-proofness*, and *individual rationality* [no agent is worse off after trading with other agents] (Ma, 1994). However, this compatibility result does not extend to "multiple object" exchange markets (Sönmez, 1999; Klaus and Miyagawa, 2002). Some recent studies for exchange markets with indivisibilities and multiple assignment problems without endowments that consider *strategy-proofness* in combination with other properties are Ehlers and Klaus (2003), Klaus and Miyagawa (2002), and Pápai (2003, 2004).

We are interested in *efficient* and *individually rational* exchange rules. In addition, we do not want any trader to be able to successfully manipulate the outcome to her advantage by hiding or destroying part of her endowment or transferring part of it to another trader who is not worse off because of the transfer.[1] We call an exchange rule that is immune to this type of manipulation *hiding-proof*, *destruction-proof*, and *transfer-proof*, respectively.

In the context of classical exchange economies, Postlewaite (1979) is the first to introduce and study *hiding-proofness* and *destruction-proofness*. He shows that, when preferences are continuous, strictly increasing, and strictly convex, *hiding-proofness* is incompatible with *efficiency* and *individual rationality*. He also shows that *destruction-proofness* is compatible with *efficiency* and *individual rationality*.[2] For reallocation problems with single-peaked preferences, Klaus, Peters, and Storcken (1997) consider *hiding-proof* rules satisfying various fairness and/or consistency properties. In the context of two-sided matching with endowments, Sertel and Özkal-Sanver (2002) and Fiestras-Janeiro, Klijn, and Sánchez (2004) analyze the manipulability of men- (women-) optimal matching rules via endowments (their non-manipulability by predonation corresponds to our *transfer-proofness* condition). *Transfer-proofness* is also related to the so-called "transfer paradox" (a trader can be hurt by accepting a predonation). Leontief (1936) is the first to demonstrate that the Walrasian rule is not immune to the transfer paradox for two-agent exchange economies. For two-agent economies, *transfer-proofness* is equivalent to being immune to the transfer-paradox. Thomson (1987) shows that *transfer-proofness* is compatible with *efficiency* and *individual rationality* in exchange economies.

We demonstrate that, similarly as in other models, *efficient* and *individually rational* rules are generally not immune to manipulations via endowments (Theorems 1, 2, and 3). However, we also identify some subclasses of exchange markets where these incompatibilities do not apply: for two agents with separable and responsive preferences, *destruction-proofness* is compatible with *efficiency* and *individual rationality* (Proposition 1), and for two agents with unrestricted preferences, *efficiency* is stronger than *transfer-proofness* so that *transfer-proofness* is compatible with *efficiency* and *individual rationality*.[3]

[1] Alternatively, we consider myopic transfer-proofness by requiring that the recipient of the transfer experiences the transfer as weakly endowment improving.

[2] Thomson (1987) strengthens the former result by showing that the incompatibility persists on the restricted domain of homothetic preferences even if *hiding-proofness* is replaced by a weaker notion at which agents can consume only a positive percentage of what they hide no matter how small that percentage is.

[3] See also Proposition 2 in the Apendix for the compatibility of *myopic transfer-proofness* with *efficiency* and *individual rationality*.

2 The Model

2.1 Exchange Markets with Indivisible Objects

Let K be a *set of heterogeneous objects* containing at least two objects (we allow $|K| = \infty$). Let 2^K denote the set of all (possibly empty) subsets of K. To simplify notation, we omit the brackets when denoting subsets of K and write, for instance, xyz instead of $\{x, y, z\}$. Let $N \equiv \{1, \ldots, n\}$ be a *finite set of agents* containing at least two agents. Each agent $i \in N$ is endowed with a finite (possibly empty) set of objects $E_i \in 2^K$. No two agents own the same object(s). So, an *endowment distribution* $E \equiv (E_1, \ldots, E_n)$ is defined by **(i)** for all $i \in N$, $|E_i| < \infty$, **(ii)** $\bigcup_{i=1}^n E_i \in 2^K$, and **(iii)** for all $i, j \in N$, $E_i \cap E_j = \emptyset$ if $i \neq j$. Note that $\bigcup_{i=1}^n E_i \subsetneq K$ is possible. We denote the *set of all endowment distributions* by $\mathcal{E}$.

Each agent $i \in N$ has complete and transitive preferences R_i over 2^K. The associated strict preference relation is denoted by P_i. Moreover, preferences are strict, that is, for all distinct subsets $S, S' \in 2^K$, either $S \; P_i \; S'$ or $S' \; P_i \; S$. Thus, $S \; R_i \; S'$ means that either $S \; P_i \; S'$ or $S = S'$.

An important preference restriction is *separability*:[4] agent i's preferences are *separable* whenever she prefers x to $\emptyset$ if and only if for any set S not containing x she prefers $S \cup x$ to S: for all $S \subseteq K$ and all $x \in K \setminus S$, $x \; P_i \; \emptyset \Leftrightarrow (S \cup x) \; P_i \; S$. Together with strictness and completeness of preferences, this implies that for all $S \subseteq K$ and all $x \in K \setminus S$, $\emptyset \; P_i \; x \Leftrightarrow S \; P_i \; (S \cup x)$. Let $\mathcal{R}_s$ be the set of separable preference relations over 2^K.

A preference restriction that is often combined with separability is *responsiveness*: agent i's preferences are *responsive* if, for any two sets that differ only in one object, agent i prefers the set containing the more preferred object: for all $S \subseteq K$ and all $x, y \in K \setminus S$, $x \, P_i \, y \Rightarrow (S \cup x) \, P_i \, (S \cup y)$. Roth (1985) introduces this notion of responsiveness for college admission problems.

The last preference restriction we consider is *additivity*: agent i's preferences are *additive* if there exists a function $u_i \colon K \to \mathbb{R}$ such that for all $S, S' \in 2^K$, $S \; R_i \; S' \Leftrightarrow \sum_{k \in S} u_i(k) \geq \sum_{k \in S'} u_i(k)$.

At various points, we consider the following four domains of preferences: the (otherwise) unrestricted domain of all strict preferences $\mathcal{R}_u$; the domain of separable preferences $\mathcal{R}_s$; the domain of separable and responsive preferences $\mathcal{R}_{sr}$; and the domain of additive preferences $\mathcal{R}_a$. Clearly, $\mathcal{R}_a \subsetneq \mathcal{R}_{sr} \subsetneq \mathcal{R}_s \subsetneq \mathcal{R}_u$. Whenever we introduce notation or concepts that apply to all preference domains, we use the generic preference domain $\mathcal{R}$. We denote a typical *preference profile* by $R = (R_1, R_2, \ldots, R_n)$ and the *set of preference profiles* by $\mathcal{R}^N$.

Thus, given a preference profile $R \in \mathcal{R}^N$ and an endowment distribution $E \in \mathcal{E}$, we denote an *exchange market (with indivisible objects)* by (R, E). Since in the remainder of the article we assume that the preference profile remains fixed while endowment distributions may vary, we simply denote an exchange market by its endowment distribution $E \in \mathcal{E}$.

An *allocation* for an exchange market $E \in \mathcal{E}$ is a list $(S_1, \ldots, S_n)$ such that **(i)** each agent $i \in N$ receives some subset $S_i \subseteq \bigcup_{i=1}^n E_i$ and **(ii)** no two agents receive the same object: for all $i, j \in N$, $S_i \cap S_j = \emptyset$ if $i \neq j$. We allow for *free disposal*, that is, $\bigcup_{i=1}^n S_i \subsetneq \bigcup_{i=1}^n E_i$ is possible. Most of our results remain valid without free disposal (Lemma 1 is the exception).

[4] For the notion of separability we use here, we refer to Barberà, Sonnenschein and Zhou (1991).

2.2 Exchange Rules and their Properties

An *(exchange) rule* is a function φ that associates with each exchange market $E \in \mathcal{E}$ an allocation $\varphi(E) = (S_i)_{i \in N}$. Given $i \in N$, we call $\varphi_i(E)$ the *allotment* of agent i at $\varphi(E)$.

Recall that in our model preferences are fixed. In addition, we will not consider any properties that link exchange markets on the basis of preference profiles. Therefore rules are only defined with respect to the given and fixed preference profile. In particular, when introducing dictatorial rules (see Examples 2–6 and 8–11) this means that the corresponding "dictatorial structure" may change across preference profiles.

Two standard requirements for rules are efficiency and individual rationality (agents find their allotments at least as good as their endowments):

Efficiency: For all $E \in \mathcal{E}$ there is no allocation $(S_i)_{i \in N}$ such that for all $i \in N$, $S_i R_i \varphi_i(E)$, with strict preference holding for some $j \in N$.

Individual Rationality: For all $E \in \mathcal{E}$ and all $i \in N$, $\varphi_i(E)\ R_i\ E_i$.

For all $E \in \mathcal{E}$, we denote the *set of efficient allocations* by $\mathcal{P}(E)$, the *set of individually rational allocations* by $\mathcal{I}(E)$, and the *set of efficient and individually rational allocations* by $\mathcal{PI}(E)$.

Given that individual endowments are private information, an agent may manipulate the outcome to her advantage by hiding, destroying, or transferring part of her endowment.

Given an endowment distribution $E \in \mathcal{E}$, an agent $i \in N$, and a subset $E'_i \subsetneq E_i$, we obtain the new endowment distribution (E'_i, E_{-i}) where agent i hides part of her endowment by replacing agent i's endowment E_i with E'_i.

First, we consider *hiding-proofness*: if agent i hides part of her endowment E_i and pretends to only own $E'_i \subsetneq E_i$, then she finds her original allotment $\varphi_i(E)$ at least as good as the set of objects $\varphi_i(E'_i, E_{-i}) \cup (E_i \backslash E'_i)$ she finally can consume.

Hiding-Proofness: For all $E \in \mathcal{E}$, all $i \in N$, and all $E'_i \subsetneq E_i$, $\varphi_i(E)\ R_i\ [\varphi_i(E'_i, E_{-i}) \cup (E_i \backslash E'_i)]$.

Since an agent could hide all of her endowment ($E'_i = \emptyset$), we deduce the following:

Lemma 1. *For any profile of separable preferences, efficiency and hiding-proofness together imply individual rationality.*[5]

Proof: Let φ be *efficient* and *hiding-proof*. Let $E \in \mathcal{E}$ and $i \in N$. If $\varphi_i(\emptyset, E_{-i}) = \emptyset$, then by *hiding-proofness*, $\varphi_i(E) R_i (\varphi_i(\emptyset, E_{-i}) \cup E_i) = E_i$. If $\varphi_i(\emptyset, E_{-i}) = \{x_1, \ldots, x_{l-1}, x_l\} \neq \emptyset$, then by *efficiency*, separability, strictness, and free disposal, for all $k \in \{1, \ldots, l\}$, $x_k\ P_i\ \emptyset$. By separability, $(\varphi_i(\emptyset, E_{-i}) \cup E_i)\ P_i\ (\{x_1, \ldots, x_{l-1}\} \cup E_i)\ P_i \ldots P_i\ (x_1 \cup E_i)\ P_i\ E_i$. Hence, by transitivity, $(\varphi_i(\emptyset, E_{-i}) \cup E_i)\ P_i\ E_i$. By *hiding-proofness*, $\varphi_i(E)\ R_i\ (\varphi_i(\emptyset, E_{-i}) \cup E_i)$. Thus, by transitivity, $\varphi_i(E)\ P_i\ E_i$. To summarize, for all $E \in \mathcal{E}$ and $i \in N$, $\varphi_i(E)\ R_i\ E_i$, *i.e.*, φ is *individually rational*. □

If each object is *desirable* to each agent, that is, for all $i \in N$, and all $x \in K$, $x P_i \emptyset$, then Lemma 1 holds without *efficiency* (that is, *hiding-proofness* implies *individual rationality*). If each object is desirable to each agent, separability is equivalent to *monotonicity*, that is, for all $i \in N$, and all $S, S' \in 2^K$, if $S \supsetneq S'$, then $S\ P_i\ S'$. In fact, "*hiding-proofness* implies *individual rationality*" is a model-free observation if preferences are monotonic.

[5]Special thanks to Somdeb Lahiri for pointing out with an example that Lemma 1 is not true on $\mathcal{R}_u$.

Lemma 1 may not be valid without free disposal (*e.g.*, any *efficient* and *hiding-proof* rule for the free disposal setting can be easily extended to the "no-disposal" setting by assigning undesirable objects to a predetermined agent).

Second, we consider *destruction-proofness*: if an agent i destroys part of her endowment E_i, thereby reducing it to $E'_i \subsetneq E_i$, then she finds her original allotment $\varphi_i(E)$ at least as good as her new allotment $\varphi_i(E'_i, E_{-i})$.

Destruction-Proofness: For all $E \in \mathcal{E}$, all $i \in N$, and all $E'_i \subsetneq E_i$, $\varphi_i(E)\, R_i\, \varphi_i(E'_i, E_{-i})$.

Given an endowment distribution $E \in \mathcal{E}$, agents $i, j \in N$, and a subset $E'_i \subsetneq E_i$, we obtain the new endowment distribution (E'_i, E'_j, E_{-ij}) where agent i transfers part of her endowment, namely $E_i \backslash E'_i$, to agent j by replacing agent i's endowment E_i with E'_i and agent j's endowment E_j with $E'_j \equiv E_j \cup E_i \backslash E'_i$. We denote the exchange market that is obtained after agent i transfers $E_i \backslash E'_i$ to agent j by (E'_i, E'_j, E_{-ij}).

Third, we consider *(farsighted) transfer-proofness*: if agent i transfers part of her endowment E_i to another agent, say agent j, who is not worse off because of the transfer, *i.e.*, $\varphi_j(E'_i, E'_j, E_{-ij})\, R_j\, \varphi_j(E)$, then agent i finds her original allotment $\varphi_i(E)$ at least as good as her new allotment $\varphi_i(E'_i, E'_j, E_{-ij})$.

Transfer-Proofness: For all $E \in \mathcal{E}$, all $i, j \in N$, all $E'_i \subsetneq E_i$, and $E'_j \equiv E_j \cup E_i \backslash E'_i$, if $\varphi_j(E'_i, E'_j, E_{-ij})\, R_j\, \varphi_j(E)$, then $\varphi_i(E)\, R_i\, \varphi_i(E'_i, E'_j, E_{-ij})$.

Obviously, *efficiency* implies *transfer-proofness* if $n = 2$. Note that we only require that the recipient of the transfer experiences it as weakly allotment improving. We do not require that the recipient (weakly) prefers her endowment after the transfer to her endowment before the transfer. Hence, in the definition of *transfer-proofness*, we assume transfer recipients to be farsighted. By imposing the extra condition that transfer recipients experience the transfer as weakly endowment improving, we obtain the following weaker transfer-proofness property.

Weak Transfer-Proofness: For all $E \in \mathcal{E}$, all $i, j \in N$, all $E'_i \subsetneq E_i$, and $E'_j \equiv E_j \cup E_i \backslash E'_i$, if $E'_j\, R_j\, E_j$ and $\varphi_j(E'_i, E'_j, E_{-ij})\, R_j\, \varphi_j(E)$, then $\varphi_i(E)\, R_i\, \varphi_i(E'_i, E'_j, E_{-ij})$.

Obviously, *transfer-proofness* implies *weak transfer-proofness*. Note that if we only impose that transfer recipients experience the transfer as weakly endowment improving, we obtain a "*myopic transfer-proofness*" condition that is logically independent from our farsighted transfer-proofness conditions. All our results remain valid if we use *myopic transfer-proofness* instead of *(farsighted) transfer-proofness*. In the Appendix, we formally define *myopic transfer-proofness* and show how examples and proofs adapt if *transfer proofness* is replaced by *myopic transfer-proofness*.

As the following examples demonstrate, no direct relationship exists between *hiding-proofness*, *destruction-proofness*, and *(weak) transfer-proofness*.

Example 1. *No-Trade Rule*

For any preference profile, the no-trade rule, a rule that assigns to each agent her endowment, is *hiding-proof* and *individually rational*. However, even for profiles with additive preferences, the no-trade rule may be neither *destruction-proof*, nor *weakly transfer-proof*, nor *efficient*. If $n = 2$, then for any preference profile, the no-trade rule is *transfer-proof* as well.

Since later we show that no *efficient* and *hiding-proof* rule exists, it is not possible to find a rule that is *efficient*, *hiding-proof*, but not *destruction-proof* or not *weakly transfer-proof*. $\diamond$

Example 2. ***Serial Dictatorship Rule***
For any profile of separable preferences, a serial dictatorship rule, a rule that assigns to each agent in a serial way her most preferred set of objects (among the remaining objects), is *destruction-proof*, *transfer-proof*, and *efficient*. However, even for profiles with additive preferences, a serial dictatorship rule may be neither *hiding-proof*, nor *individually rational*.

We refer to Klaus and Miyagawa (2002) for a precise definition of serial dictatorship rules. For unrestricted preference profiles, a serial dictatorship rule may not be *destruction-proof* (*e.g.*, destroying an object may cause a predecessor to abstain from consuming other objects that she considers complementary to the destroyed one). $\diamond$

Example 3. ***Conditional Serial Dictatorship Rule*** $\varphi^{csd(x,E)}$
A conditional serial dictatorship rule $\varphi^{csd(x,E)}$ is defined as follows: Let $x \in K$ and φ^d, $\varphi^{d'}$ be serial dictatorship rules such that for φ^d, lower-indexed agents come first and for $\varphi^{d'}$, higher-indexed agents come first. For all $E \in \mathcal{E}$ such that $x \in \bigcup_{i \in N} E_i$, let $\varphi^{csd(x,E)}(E) \equiv \varphi^d(E)$. For all $E \in \mathcal{E}$ such that $x \notin \bigcup_{i \in N} E_i$, let $\varphi^{csd(x,E)}(E) \equiv \varphi^{d'}(E)$.

For any profile of separable preferences, $\varphi^{csd(x,E)}$ is *efficient* and *transfer-proof*. However, even for profiles with additive preferences, $\varphi^{csd(x,E)}$ may be neither *hiding-proof*, nor *individually rational*, nor *destruction-proof*. $\diamond$

Example 4. ***Conditional Serial Dictatorship Rule*** $\varphi^{csd(x,E_1)}$
A conditional serial dictatorship rule $\varphi^{csd(x,E_1)}$ is defined as follows: Let $x \in K$ and φ^d, $\varphi^{d'}$ be serial dictatorship rules such that for φ^d, lower-indexed agents come first and for $\varphi^{d'}$, higher-indexed agents come first. For all $E \in \mathcal{E}$ such that $x \in E_1$, let $\varphi^{csd(x,E_1)}(E) \equiv \varphi^d(E)$. For all $E \in \mathcal{E}$ such that $x \notin E_1$, let $\varphi^{csd(x,E_1)}(E) \equiv \varphi^{d'}(E)$.

Let $n \geq 3$. Then, for any profile of separable preferences, $\varphi^{csd(x,E_1)}$ is *efficient* and *destruction-proof*. However, even for profiles with additive preferences, $\varphi^{csd(x,E_1)}$ may be neither *hiding-proof*, nor *individually rational*, nor *weakly transfer-proof*. If $n = 2$, then for any profile of separable preferences, $\varphi^{csd(x,E_1)}$ is *transfer-proof* as well. $\diamond$

3 Results

3.1 Hiding-Proofness

Theorem 1. *For some profiles of additive preferences, no rule is efficient and hiding-proof.*

Proof: Let φ be an *efficient* and *hiding-proof* rule. Let $N = \{1, 2\}$, $E = (E_1, E_2)$ be such that $E_1 = ab$, $E_2 = cd$, and $(R_1, R_2) \in \mathcal{R}_a^N$ have the following utility representation

$u_1(a) = 5,$	$u_2(a) = 6,$
$u_1(b) = 2.1,$	$u_2(b) = 3,$
$u_1(c) = 3,$	$u_2(c) = 1.1,$
$u_1(d) = 4,$	$u_2(d) = 4.$

Hence, by Lemma 1, φ is *individually rational*. The only *efficient* and *individually rational* allocations are $A = (ac, bd)$ and $B = (bcd, a)$. Hence, $\varphi(E) \in \{A, B\}$.

Case 1: $\varphi(E) = A$. If agent 1 hides object b, the endowment distribution becomes $E^1 = (a, cd)$ and the only *efficient* and *individually rational* allocation for the resulting exchange market is $A^1 = (cd, a)$. So, $\varphi(E^1) = A^1$. Hence, agent 1 consumes bcd, which she prefers to ac, her allotment at A, in violation of *hiding-proofness*. Thus, $\varphi(E) \neq A$.

Case 2: $\varphi(E) = B$. If agent 2 hides object d, the endowment distribution becomes $E^2 = (ab, c)$ and the only *efficient* and *individually rational* allocation for the resulting exchange market is $B^1 = (ad, b)$. So, $\varphi(E^2) = B^1$. Hence, agent 2 consumes bd, which she prefers to bc, her allotment at B, in violation of *hiding-proofness*. Thus, $\varphi(E) \neq B$.

Cases 1 and 2 together show that for $n = 2$, *efficiency* and *hiding-proofness* are incompatible. For $n > 2$, we simply add agents who prefer their endowments to any other set of objects (including $\emptyset$). Since then only agents 1 and 2 trade with each other as specified above, the incompatibility of *efficiency* and *hiding-proofness* persists for $n > 2$. □

3.2 Destruction-Proofness

If we replace *hiding-proofness* by *destruction-proofness*, compatibility with *efficiency* and *individual rationality* is possible for two agents with separable and responsive preferences.

Let $N = \{1, 2\}$, $R \in \mathcal{R}^N_{sr}$, and $E \in \mathcal{E}$. In order to present a rule satisfying the properties listed above, we introduce some notation. First, for $i \in N$, we obtain $\bar{E}_i$ by discarding each *undesirable object* x, that is, an object $x \in E_i$ such that $\emptyset\, P_i\, x$. Second, in order to preserve *efficiency*, we define the set $\tilde{E}_i$ by adding to $\bar{E}_i$ all objects that agent $j \neq i$ discarded, and that agent i likes, that is, $\tilde{E}_i \equiv \bar{E}_i \cup \{x \in E_j \backslash \bar{E}_j : x\, P_i\, \emptyset\}$. Note that $\mathcal{PI}(\tilde{E}) \subseteq \mathcal{PI}(E)$.

Example 5. ***Restricted (Serial) Dictatorship Rule***[6] $\varphi^{rd(i)}$
Let $N = \{1, 2\}$ and $i \in N$. For all $R \in \mathcal{R}^N_s$ and all $E \in \mathcal{E}$, $\varphi^{rd(i)}$ picks the unique best allocation for agent i in $\mathcal{PI}(\tilde{E})$. We call agent i the *restricted dictator*. By construction, $\varphi^{rd(i)}$ is *efficient* and *individually rational*. ◇

Next, we show that when preferences are separable and responsive, $\varphi^{rd(i)}$ is *destruction-proof*. One can easily show that $\varphi^{rd(i)}$ is not *hiding-proof*.

Proposition 1. *For two agents with separable and responsive preferences, restricted dictatorship rules are destruction-proof.*

Proposition 1 only remains valid on $\mathcal{R}_a$ and $\mathcal{R}_{sr}$, but not on $\mathcal{R}_s$ and $\mathcal{R}_u$ (see Theorem 2). For $\mathcal{R}_u$, it is easy to see that destroying an object which is considered complementary by a previous restricted dictator, may induce this restricted dictator to choose a trade that is more advantageously for the agent who destroyed the object.

Proof: Let $N = \{1, 2\}$, $\varphi = \varphi^{rd(1)}$, $R \in \mathcal{R}^N_{sr}$, and $E \in \mathcal{E}$. Note that by definition, no agent i can benefit by destroying an undesirable object $x \in E_i$. Hence, it is without loss of

[6]For $n > 2$ we can define *restricted serial dictatorship rules* $\tilde{\varphi}^{rd(\pi)}$, where π denotes the ordering of "dictators." Similarly as before, we can derive an exchange market $\tilde{E}$ by first letting all agents discard of undesirable objects and then distributing them among the agents who would like to consume them (this distribution can, for instance, be done sequentially using π). Then, for all $R \in \mathcal{R}^N_s$ and $E \in \mathcal{E}$, the first dictator restricts the set $\mathcal{PI}(\tilde{E})$ to all allocations where she receives her best allotment. Next, if several allocations are left over, the second dictator restricts the remaining set to all allocations where she receives her best allotment, *etc.*. In order to adjust restricted serial dictatorship rules if free disposal is not allowed, we simply assume that one of the agents has to keep any object that is undesirable for all agents.

generality to assume that $E = \tilde{E}$. We prove that neither agent can benefit from destroying one of her objects. The proof that neither agent can benefit from destroying several objects follows by applying the "one-object-argument" for each object and invoking transitivity of preferences.

Case 1: Agent 1 destroys $x \in E_1$. Let $A \equiv \varphi(E)$ and $B \equiv \varphi(E_1 \backslash x, E_2)$. Suppose $B_1 P_1 A_1$. By separability, $(B_1 \cup x)\, P_1\, B_1$ and $(B_1 \cup x, B_2) \in \mathcal{I}(E)$. Hence, there exists $C \in \mathcal{PI}(E)$ such that $C_1\, R_1\, (B_1 \cup x)$. Thus, $C_1\, P_1\, A_1$, which contradicts the assumption that A is the best allocation for agent 1 in $\mathcal{PI}(E)$.

Case 2: Agent 2 destroys $x \in E_2$. Let $A \equiv \varphi(E)$ and $B \equiv \varphi(E_1, E_2 \backslash x)$. Suppose $B_2\, P_2\, A_2$. If $x \in A_2$, then $A_2\, R_2\, E_2$, which together with responsiveness implies that $A_2 \backslash x\, R_2\, E_2 \backslash x$. Then, $A \in \mathcal{PI}(E)$ implies $(A_1, A_2 \backslash x) \in \mathcal{PI}(E_1, E_2 \backslash x)$. Thus, by the definition of φ, $B_1\, R_1\, A_1$. This and $B_2\, P_2\, A_2$ contradict that $A \in \mathcal{P}(E)$. Hence, $x \in A_1$. Since $A \in \mathcal{P}(E)$, $A_1\, P_1\, (B_1 \cup x)$. By responsiveness, $A_1 \backslash x\, P_1\, B_1$. Note that $(A_1 \backslash x, A_2) \in \mathcal{I}(E_1, E_2 \backslash x)$. Hence, there exists $C \in \mathcal{PI}(E_1, E_2 \backslash x)$ such that $C_1\, P_1\, B_1$, which contradicts the assumption that B is the best allocation for agent 1 in $\mathcal{PI}(E_1, E_2 \backslash x)$. □

The following example describes a class of rules that are all *efficient*, *individually rational*, and *destruction-proof*.

Example 6. ***Restricted Conditional Dictatorship Rule*** $\varphi^{rcd(K', \bar{E}_1)}$
Let $N = \{1, 2\}$ and $K' \subseteq K$. For all $R \in \mathcal{R}_{sr}^N$ and all $E \in \mathcal{E}$ such that $K' \subseteq \bar{E}_1$, $\varphi^{rcd(K', \bar{E}_1)}(E) \equiv \varphi^{rd(1)}(E)$. For all $R \in \mathcal{R}_{sr}^N$ and all $E \in \mathcal{E}$ such that $K' \nsubseteq \bar{E}_1$, $\varphi^{rcd(K', \bar{E}_1)}(E) \equiv \varphi^{rd(2)}(E)$. Then, $\varphi^{rcd(K', \bar{E}_1)}$ is *efficient*, *individually rational*, and *destruction-proof*. ◇

Many other restricted conditional dictatorship rules that are *destruction-proof* and are similar to those in Example 6 can be constructed. For instance, one can condition the choice of the restricted dictator differently, *e.g.*, by $K' \cap \bar{E}_1 \neq \emptyset$ instead of $K' \subseteq \bar{E}_1$. Hence, the class of rules that are *efficient*, *individually rational*, and *destruction-proof* for two agents with separable and responsive preferences is very large.

The next example demonstrates that for more than two agents, a restricted serial dictatorship rule may be manipulable by destruction. This result holds for any subdomain of $\mathcal{R}_s$ that includes the domain of additive preferences $\mathcal{R}_a$, in particular, for $\mathcal{R}_a$, $\mathcal{R}_{sr}$, and $\mathcal{R}_s$ (recall that our definition of a restricted serial dictatorship rules only applies to separable preferences so that we cannot make any statements about $\mathcal{R}_u$).

Example 7. Let $N = \{1, 2, 3\}$, $E = (E_1, E_2, E_3)$ be such that $E_1 = a$, $E_2 = bc$, $E_3 = de$, and $(R_1, R_2, R_3) \in \mathcal{R}_a^N$ have the following utility representation

$u_1(a) = 1,$	$u_2(a) = 5,$	$u_3(a) = 7,$
$u_1(b) = 8,$	$u_2(b) = 4,$	$u_3(b) = 6,$
$u_1(c) = 5,$	$u_2(c) = 2,$	$u_3(c) = 1.1,$
$u_1(d) = 10.5,$	$u_2(d) = 8,$	$u_3(d) = 3,$
$u_1(e) = 0.1,$	$u_2(e) = 1.5,$	$u_3(e) = 2.3.$

If agent 1 is the restricted dictator, then the restricted serial dictatorship rule picks (cd, ae, b). However, if agent 3 destroys object e, for the resulting exchange market, the restricted serial dictatorship rule picks (bc, d, a). Hence, agent 3 consumes a, which she strictly prefers to b, in violation of *destruction-proofness*. ◇

It is an open question whether for more than two agents with additive, or separable and responsive preferences, *efficient*, *individually rational*, and *destruction-proof* rules exist. If preferences are "only" separable, then we can establish the incompatibility of *efficiency*, *individual rationality*, and *destruction-proofness* for any number of agents.

Theorem 2. *For some profiles of separable preferences, no rule is efficient, individually rational, and destruction-proof.*

Proof: Let φ be an *efficient*, *individually rational*, and *destruction-proof* rule. Let $N = \{1, 2\}$, $E = (E_1, E_2)$ be such that $E_1 = ab$, $E_2 = cde$, and $(R_1, R_2) \in \mathcal{R}_s^N$ be as in Table 1. The underline symbol '_' in each allotment indicates that the corresponding object is not in the allotment. This presentation makes it easier to verify that both preference relations are separable. In order to save space, each linear ordering is listed in two columns. Once the reader reaches the bottom of the first column, she should continue from the top of the second column. The important entries are marked in boldface and the endowments are underlined.[7]

R_1										R_2									
a	b	c	d	e	_	b	c	_	e	a	b	c	d	e	$\boldsymbol{a}$	$\boldsymbol{b}$	_	_	_
a	b	c	d	_	_	b	_	d	e	a	b	c	d	_	a	_	c	_	_
a	b	c	_	e	a	_	_	d	_	a	b	c	_	e	a	_	_	d	_
a	b	_	d	e	a	_	_	_	e	a	b	_	d	e	a	_	_	_	e
a	_	c	d	e	a	_	_	_	_	a	_	c	d	e	$\boldsymbol{a}$	_	_	_	_
_	$\boldsymbol{b}$	$\boldsymbol{c}$	$\boldsymbol{d}$	$\boldsymbol{e}$	_	b	c	_	_	_	b	c	d	e	_	_	$\underline{\boldsymbol{c}}$	$\underline{\boldsymbol{d}}$	$\underline{\boldsymbol{e}}$
_	_	$\boldsymbol{c}$	$\boldsymbol{d}$	$\boldsymbol{e}$	_	b	_	d	_	a	b	c	_	_	_	b	c	_	_
a	b	c	_	_	_	b	_	_	e	a	b	_	d	_	_	b	_	_	e
a	b	_	d	_	_	_	c	d	_	a	b	_	_	e	_	_	c	d	_
a	b	_	_	e	_	_	c	_	e	a	_	c	d	_	_	_	c	_	e
a	_	c	d	_	_	_	_	d	e	a	_	c	_	e	_	_	_	d	e
$\boldsymbol{a}$	_	$\boldsymbol{c}$	_	$\boldsymbol{e}$	_	b	_	_	_	a	_	_	d	e	_	b	_	_	_
$\boldsymbol{a}$	_	$\boldsymbol{c}$	_	_	_	_	c	_	_	_	b	c	d	_	_	_	c	_	_
$\underline{\boldsymbol{a}}$	$\underline{\boldsymbol{b}}$	_	_	_	_	_	_	d	_	_	b	c	_	e	_	_	_	d	_
a	_	_	d	e	_	_	_	_	e	_	$\boldsymbol{b}$	_	$\boldsymbol{d}$	$\boldsymbol{e}$	_	_	_	_	e
_	b	c	d	_	_	_	_	_	_	_	$\boldsymbol{b}$	_	$\boldsymbol{d}$	_	_	_	_	_	_

Table 1: Complete separable preferences in the proof of Theorem 2.

We now explain how one can reduce the preference table to allotments that can occur at *efficient* and *individually rational* allocations. First, by *individual rationality*, for each agent, we can delete all allotments that are ranked below her endowment. Next, by *individual rationality*, agent 1 has to receive at least two objects and, agent 2 has to receive a or at least two objects. Hence, we can delete all allotments containing more than three objects, except $bcde$ for agent 1. Finally, at any *efficient* allocation all objects must be assigned. Thus, we can delete allotments from an agent's preference relation if the remaining objects are not *individually rational* for the other agent. For example, since bc

[7]Note that the ranking of boldfaced entries in Table 1 contains the information on preferences we use in the proof. We then constructed Table 1 as a separable extension of these preferences.

is not *individually rational* for agent 1, agent 2 will never receive ade and we can delete the associated entry in Table 1. Hence, by *efficiency* and *individual rationality*, we can focus on the part of the preferences depicted in Table 2.

R_1					R_2				
_	b	c	d	e	_	b	_	d	e
_	_	c	d	e	_	b	_	d	_
a	_	c	_	e	a	b	_	_	_
a	_	c	_	_	a	_	_	_	_
a	b	_	_	_	_	_	c	d	e

Table 2: Relevant entries in Table 1 after taking *efficiency* and *individual rationality* into account.

Note that E is not *efficient*. Hence, according to Table 2, the only *efficient* and *individually rational* allocations are $A = (bcde, a)$, $B = (cde, ab)$, $C = (ace, bd)$, and $D = (ac, bde)$. Hence, $\varphi(E) \in \{A, B, C, D\}$.

Case 1: $\varphi(E) \in \{A, B\}$. If agent 2 destroys object e, the endowment distribution becomes $E^1 = (ab, cd)$. It is easy to check that the only *efficient* and *individually rational* allocation for the resulting exchange market is $A^1 = (ac, bd)$. So, $\varphi(E^1) = A^1$. Hence, agent 2 consumes bd, which she prefers to a, her allotment at A; and to ab, her allotment at B, in violation of *destruction-proofness*. Thus, $\varphi(E) \notin \{A, B\}$.

Case 2: $\varphi(E) \in \{C, D\}$. If agent 1 destroys object b, the endowment distribution becomes $E^2 = (a, cde)$. It is easy to check that the only *efficient* and *individually rational* allocation for the resulting exchange market is $C^1 = (cde, a)$. So, $\varphi(E^2) = C^1$. Hence, agent 1 consumes cde, which she prefers to ace, her allotment at C; and to ac, her allotment at D, in violation of *destruction-proofness*. Thus, $\varphi(E) \notin \{C, D\}$.

Cases 1 and 2 together show that for $n = 2$, *efficiency*, *individual rationality*, and *destruction-proofness* are incompatible. For $n > 2$, we simply add agents who prefer their endowments to any other set of objects (including $\emptyset$). Since then only agents 1 and 2 trade with each other as specified above, the incompatibility of *efficiency*, *individual rationality*, and *destruction-proofness* persists for $n > 2$. □

3.3 Transfer-Proofness

For two agents, *efficiency* implies *transfer-proofness*. Hence, when $n = 2$, for any preference profile, any *efficient* and *individually rational* rule is *transfer-proofness*.

It is an open question whether for more than two agents with either additive, or separable and responsive preferences, *efficient*, *individually rational*, and *weakly transfer-proof* rules exist. However, for more than two agents with separable preferences, these properties are not compatible.

Theorem 3. *For some profiles of separable preferences and at least three agents, no rule is efficient, individually rational, and weakly transfer-proof.*

Proof: Let φ be an *efficient*, *individually rational*, and *transfer-proof* rule. Let $N = \{1, 2, 3\}$, $E = (E_1, E_2, E_3)$ be such that $E_1 = ab$, $E_2 = cd$, $E_3 = ef$, and $(R_1, R_2, R_3) \in \mathcal{R}_s^N$ be as in Table 3 (the structure is as in Table 1). The important entries are marked in bold face and the endowments are underlined.[8]

R_1		R_2		R_3	
$a~b~c~d~e~f$	$\_~\_~c~\_~e~f$	$a~b~c~d~e~f$	$a~\_~\_~\_~e~f$	$a~b~c~d~e~f$	$\_~b~\_~d~e~f$
$a~b~c~d~\_~f$	$\_~\_~\_~\_~\boldsymbol{e}~\boldsymbol{f}$	$a~b~c~d~\_~f$	$\boldsymbol{a}~\_~\_~\_~\boldsymbol{e}~\_$	$a~b~c~d~e~\_$	$a~\_~\_~\_~e~f$
$a~b~\_~d~e~f$	$a~b~c~d~\_~\_$	$a~b~c~\_~e~f$	$\_~b~c~d~e~\_$	$a~b~c~d~\_~f$	$\_~b~\_~\_~e~f$
$a~\_~c~d~e~f$	$a~b~c~\_~\_~f$	$a~b~\_~d~e~f$	$\_~\_~c~d~e~f$	$a~\_~c~d~e~f$	$\_~\_~\_~d~e~f$
$\_~b~c~d~e~f$	$a~b~c~\_~\_~\_$	$\_~b~c~d~e~f$	$\_~b~c~d~\_~\_$	$\_~b~c~d~e~f$	$\_~\_~\_~\_~\underline{\boldsymbol{e}}~\underline{\boldsymbol{f}}$
$a~b~\_~d~\_~f$	$a~b~\_~d~\_~\_$	$a~b~c~\_~\_~f$	$\_~\_~c~d~e~\_$	$a~b~c~d~\_~\_$	$a~b~\_~d~\_~f$
$a~\_~c~d~\_~f$	$a~b~\_~\_~\_~f$	$a~b~\_~d~\_~f$	$\_~\_~c~d~\_~f$	$a~\_~c~d~e~\_$	$a~b~\_~\_~\_~f$
$a~\_~\_~d~e~f$	$\underline{\boldsymbol{a}}~\underline{\boldsymbol{b}}~\_~\_~\_~\_$	$a~b~\_~\_~e~f$	$\_~\_~\underline{\boldsymbol{c}}~\underline{\boldsymbol{d}}~\_~\_$	$a~\_~c~d~\_~f$	$a~\_~\_~d~\_~f$
$\_~b~c~d~\_~f$	$\_~b~c~d~\_~\_$	$\_~b~c~d~\_~f$	$a~\_~\_~d~\_~f$	$\_~b~c~d~e~\_$	$\_~b~\_~d~\_~f$
$\_~b~\_~d~e~f$	$\_~b~c~\_~e~\_$	$\_~b~c~\_~e~f$	$\_~b~\_~d~e~\_$	$\_~b~c~d~\_~f$	$a~\_~\_~\_~\_~f$
$\_~\_~c~d~e~f$	$\_~b~c~\_~\_~f$	$\_~b~\_~d~e~f$	$\_~\_~\_~d~e~f$	$\_~\_~c~d~e~f$	$\_~b~\_~\_~\_~f$
$a~\_~\_~d~\_~f$	$\_~b~c~\_~\_~\_$	$a~b~\_~\_~\_~f$	$a~\_~\_~d~\_~\_$	$a~\_~c~d~\_~\_$	$\_~\_~c~\_~\_~f$
$\_~b~\_~d~\_~f$	$\_~b~\_~d~\_~\_$	$\_~b~c~\_~\_~f$	$\_~b~\_~d~\_~\_$	$\_~b~c~d~\_~\_$	$\_~\_~\_~d~\_~f$
$\_~\_~c~d~\_~f$	$\_~b~\_~\_~e~\_$	$\_~b~\_~d~\_~f$	$\_~\_~\_~d~e~\_$	$\_~\_~c~d~e~\_$	$\_~\_~\_~\_~\_~\boldsymbol{f}$
$\_~\_~\_~d~e~f$	$\_~b~\_~\_~\_~f$	$\_~b~\_~\_~e~f$	$\_~\_~\_~d~\_~f$	$\_~\_~c~d~\_~f$	$a~b~\_~d~e~\_$
$\_~\_~\_~\boldsymbol{d}~\_~\boldsymbol{f}$	$\_~\boldsymbol{b}~\_~\_~\_~\_$	$\_~\boldsymbol{b}~\_~\_~\_~\boldsymbol{f}$	$\_~\_~\_~\boldsymbol{d}~\_~\_$	$\_~\_~\boldsymbol{c}~\boldsymbol{d}~\_~\_$	$a~b~\_~d~\_~\_$
$a~b~c~d~e~\_$	$a~\_~c~d~\_~\_$	$a~b~c~d~e~\_$	$a~\_~c~\_~\_~f$	$a~b~c~\_~e~f$	$a~b~\_~\_~e~\_$
$a~b~c~\_~e~f$	$a~\_~c~\_~e~\_$	$a~\_~c~d~e~f$	$\_~b~c~\_~e~\_$	$a~\_~c~\_~e~f$	$a~\_~\_~d~e~\_$
$a~b~c~\_~e~\_$	$a~\_~c~\_~\_~f$	$a~b~c~d~\_~\_$	$\_~\_~c~\_~e~f$	$\_~b~c~\_~e~f$	$\_~b~\_~d~e~\_$
$a~b~\_~d~e~\_$	$a~\_~c~\_~\_~\_$	$a~\_~c~d~e~\_$	$a~\_~c~\_~\_~\_$	$\_~\_~\boldsymbol{c}~\_~\boldsymbol{e}~\boldsymbol{f}$	$a~b~\_~\_~\_~\_$
$a~b~\_~\_~e~f$	$a~\_~\_~d~\_~\_$	$a~\_~c~d~\_~f$	$a~\_~\_~\_~\_~f$	$a~b~c~\_~e~\_$	$a~\_~\_~d~\_~\_$
$\boldsymbol{a}~\boldsymbol{b}~\_~\_~\boldsymbol{e}~\_$	$a~\_~\_~\_~e~\_$	$\boldsymbol{a}~\_~\boldsymbol{c}~\boldsymbol{d}~\_~\_$	$\_~b~c~\_~\_~\_$	$a~b~c~\_~\_~f$	$a~\_~\_~\_~e~\_$
$a~\_~c~d~e~\_$	$a~\_~\_~\_~\_~f$	$a~b~c~\_~e~\_$	$\_~b~\_~\_~e~\_$	$a~b~c~\_~\_~\_$	$\_~b~\_~d~\_~\_$
$\_~b~c~d~e~\_$	$\_~\_~c~d~\_~\_$	$a~b~\_~d~e~\_$	$\_~\_~c~\_~e~\_$	$\_~b~c~\_~e~\_$	$\_~b~\_~\_~e~\_$
$a~\_~\_~d~e~\_$	$\_~\_~c~\_~e~\_$	$a~b~c~\_~\_~\_$	$\_~\_~c~\_~\_~f$	$\_~b~c~\_~\_~f$	$\_~\_~c~\_~e~\_$
$\_~b~\_~d~e~\_$	$\_~\_~c~\_~\_~f$	$a~b~\_~d~\_~\_$	$\_~\_~\_~\_~e~f$	$\_~\boldsymbol{b}~\boldsymbol{c}~\_~\_~\_$	$\_~\_~\_~d~e~\_$
$\_~\_~c~d~e~\_$	$a~\_~\_~\_~\_~\_$	$a~b~\_~\_~e~\_$	$a~\_~\_~\_~\_~\_$	$a~\_~c~\_~e~\_$	$a~\_~\_~\_~\_~\_$
$\_~\_~\_~\boldsymbol{d}~\boldsymbol{e}~\_$	$\_~\_~c~\_~\_~\_$	$\boldsymbol{a}~\boldsymbol{b}~\_~\_~\_~\_$	$\_~b~\_~\_~\_~\_$	$a~\_~c~\_~\_~f$	$\_~b~\_~\_~\_~\_$
$a~\_~c~\_~e~f$	$\_~\_~\_~d~\_~\_$	$a~\_~c~\_~e~f$	$\_~\_~c~\_~\_~\_$	$\boldsymbol{a}~\_~\boldsymbol{c}~\_~\_~\_$	$\_~\_~c~\_~\_~\_$
$\_~b~c~\_~e~f$	$\_~\_~\_~\_~e~\_$	$a~\_~\_~d~e~f$	$\_~\_~\_~\_~e~\_$	$a~b~\_~d~e~f$	$\_~\_~\_~d~\_~\_$
$a~\_~\_~\_~e~f$	$\_~\_~\_~\_~\_~f$	$a~\_~c~\_~e~\_$	$\_~\_~\_~\_~\_~f$	$a~b~\_~\_~e~f$	$\_~\_~\_~\_~e~\_$
$\_~b~\_~\_~e~f$	$\_~\_~\_~\_~\_~\_$	$a~\_~\_~d~e~\_$	$\_~\_~\_~\_~\_~\_$	$a~\_~\_~d~e~f$	$\_~\_~\_~\_~\_~\_$

Table 3: Complete separable preferences in the proof of Theorem 3.

We now explain how one can reduce the preference table to allotments that can occur at *efficient* and *individually rational* allocations. First, by *individual rationality*, for each agent, we can delete all allotments that are ranked below her endowment. All *individually*

[8]Note that the ranking of boldfaced entries in Table 3 contains the information on preferences we use in the proof. We then constructed Table 3 as a separable extension of these preferences.

rational allotments that are left contain at least two objects for each agent. Hence, we can delete all allotments containing more than two objects. Thus, by *individual rationality*, we can focus on the part of the preferences depicted in Table 4.

R_1						R_2						R_3					
_	_	_	d	_	f	_	b	_	_	_	f	_	_	c	d	_	_
_	_	_	d	e	_	a	b	_	_	_	_	_	b	c	_	_	_
_	_	_	_	e	f	a	_	_	_	e	_	a	_	c	_	_	_
a	b	_	_	_	_	_	_	c	d	_	_	_	_	_	_	e	f

Table 4: Relevant entries in Table 3 after taking *individual rationality* into account.

Note that E is not *efficient*. Hence, according to Table 4, the only *efficient* and *individually rational* allocations are $A = (df, ae, bc)$, $B = (de, bf, ac)$, and $C = (ef, ab, cd)$. Hence, $\varphi(E) \in \{A, B, C\}$.

Case 1: $\varphi(E) = A$. If agent 2 transfers object c to agent 3, the endowment distribution becomes $E^1 = (ab, d, cef)$. We now explain how one can reduce the preference table to allotments that can occur at *efficient* and *individually rational* allocations for exchange market E^1. First, by *individual rationality*, for each agent, we can delete all allotments that are ranked below her endowment. Next, by *individual rationality*, agent 1 has to receive at least two objects; agent 2 has to keep her endowment or receive at least two objects, and agent 3 has to receive *cd* or at least three objects. Thus, agent 1 can receive three objects only if none of them are c or d, and agent 2 can receive two objects only if none of them are c or d. Hence, by *individual rationality*, we can focus on the part of the preferences depicted in Table 5.

R_1						R_2						R_3					
_	_	_	d	_	f	_	b	_	_	_	f	a	_	c	d	_	_
a	b	_	_	e	_	a	b	_	_	_	_	_	b	c	d	_	_
_	_	_	d	e	_	a	_	_	_	e	_	_	_	c	d	e	_
a	_	_	_	e	f	_	_	_	d	_	_	_	_	c	d	_	f
_	b	_	_	e	f							_	_	c	d	_	_
_	_	_	_	e	f							_	_	c	_	e	f
a	b	_	_	_	f												
a	b	_	_	_	_												

Table 5: Case 1, relevant entries in Table 3 after taking *individual rationality* into account.

Note that E^1 is not *efficient*. Hence, according to Table 5, the only *efficient* and *individually rational* allocation for the resulting exchange market is $C = (ef, ab, cd)$. So, $\varphi(E^1) = C$. Hence, agent 2 consumes *ab*, which she prefers to *ae*, her allotment at A, in violation of *transfer-proofness*. Since agent 3 receives *cd*, which she prefers to *bc*, we also have a contradiction to *weak transfer-proofness*. Thus, $\varphi(E) \neq A$.

Case 2: $\varphi(E) = B$. If agent 3 transfers object e to agent 1, the endowment distribution becomes $E^2 = (abe, cd, f)$. Using similar arguments as in Case 1, we can focus on the part of the preferences depicted in Table 6.

R_1	R_2	R_3
a _ _ d _ f	_ b _ _ _ f	_ b c _ _ _
_ b _ d _ f	a b c _ _ _	a _ c _ _ _
_ _ c d _ f	a b _ _ e _	_ _ _ _ _ f
_ _ _ d e f	a b _ _ _ _	
_ _ _ d _ f	a _ c _ e _	
a b _ _ e _	a _ _ _ e _	
	_ _ c d _ _	

Table 6: Case 2, relevant entries in Table 3 after taking *individual rationality* into account.

Note that E^2 is not *efficient*. Hence, according to Table 6, the only *efficient* and *individually rational* allocation for the resulting exchange market is $A = (df, ae, bc)$. So, $\varphi(E^2) = A$. Hence, agent 3 consumes bc, which she prefers to ac, her allotment at B, in violation of *transfer-proofness*. Since agent 1 receives df, which she prefers to de, we also have a contradiction to *weak transfer-proofness*. Thus, $\varphi(E) \neq B$.

Case 3: $\varphi(E) = C$. If agent 1 transfers object a to agent 2, the endowment distribution becomes $E^3 = (b, acd, ef)$. Using similar arguments as in Case 1, we can focus on the part of the preferences depicted in Table 7.

R_1	R_2	R_3
_ _ _ d e _	a b _ _ _ f	a _ c d _ _
_ b _ _ _ _	_ b c _ _ f	_ _ c d e _
	_ b _ d _ f	_ _ c d _ _
	_ b _ _ e f	_ b c _ _ _
	_ b _ _ _ f	a _ c _ e _
	a _ c d _ _	a _ c _ _ _
		_ _ _ _ e f

Table 7: Case 3, relevant entries in Table 3 after taking *individual rationality* into account.

Note that E^3 is not *efficient*. Hence, according to Table 7, the only *efficient* and *individually rational* allocation for the resulting exchange market is $B = (de, bf, ac)$. So, $\varphi(E^3) = B$. Hence, agent 1 consumes de, which she prefers to ef, her allotment at C, in violation of *transfer-proofness*. Since agent 2 receives bf, which she prefers to ab, we also have a contradiction to *weak transfer-proofness*. Thus, $\varphi(E) \neq C$.

Cases 1, 2, and 3 together show that *efficiency*, *individual rationality*, and *weak transfer-proofness* are incompatible for three agents. For $n > 3$, we simply add agents who prefer their endowments to any other set of objects (including $\emptyset$). Since then, only agents 1, 2, and 3 trade with each other as specified above, the incompatibility of *efficiency*, *individual rationality*, and *weak transfer-proofness* persists for $n > 3$. □

4 Conclusion

In this paper, we show that for separable preferences, *efficient* and *individually rational* rules are generally not immune to manipulations via endowments (Theorems 1, 2, and 3). An exception is the compatibility of *efficiency*, *individual rationality* and *transfer-proofness* in the two-agent case (*efficiency* then implies *transfer-proofness*). If in addition to separability we impose responsiveness, we obtain compatibility of *efficiency*, *individual rationality*, and *destruction-proofness* in the two-agent case (Proposition 1). We conjecture that the "dividing line" between compatibility and incompatibility lies between the preference domains $\mathcal{R}_{sr}$ and $\mathcal{R}_s$. However, two interesting questions we could not answer are:

Are *efficiency*, *individual rationality*, and *destruction-proofness* compatible for more than two agents with separable and responsive (additive) preferences?

Are *efficiency*, *individual rationality*, and *(weak) transfer-proofness* compatible for more than two agents with separable and responsive (additive) preferences?

A reason why it is not easy to answer these questions is that as the numbers of agents and objects become larger, it gets more difficult to determine the set of *efficient* and *individually rational* allocations.

Appendix: Myopic Transfer-Proofness

As an alternative transfer-proofness condition we now consider *myopic transfer-proofness*: if agent i transfers part of her endowment E_i to another agent, say agent j, who experiences the transfer as weakly endowment improving, thereby reducing her endowment to $E'_i \subsetneq E_i$, and expanding agent j's endowment to $E'_j \supsetneq E_j$ such that $E'_j \, R_j \, E_j$, then agent i finds her original allotment $\varphi_i(E)$ at least as good as her new allotment $\varphi_i(E'_i, E'_j, E_{-ij})$.

Myopic Transfer-Proofness: For all $E \in \mathcal{E}$, all $i, j \in N$, all $E'_i \subsetneq E_i$, and $E'_j \equiv E_j \cup E_i \backslash E'_i$, if $E'_j \, R_j \, E_j$, then $\varphi_i(E) \, R_i \, \varphi_i(E'_i, E'_j, E_{-ij})$.

Obviously, *myopic transfer-proofness* implies *weak transfer-proofness*. Furthermore, the serial dictatorship rules and conditional serial dictatorship rules $\varphi^{csd(x,E)}$ are *myopic transfer-proof* (see Examples 2 and 3). Clearly, the no-trade rule and the conditional serial dictatorship rules $\varphi^{csd(x,E_1)}$ are not *myopic transfer-proof* (see Examples 1 and 4). Hence, *myopic transfer-proofness* is logically independent of *hiding-proofness* and *destruction-proofness*.

Next, we consider the independence of *myopic transfer-proofness* and *transfer-proofness*. There are two cases of underlying separable preference profiles where *myopic transfer-proofness* and *transfer-proofness* are not logically independent.

The first case is when no object is desirable, *i.e.*, for all $x \in K$ and all $i \in N$, $\emptyset \, P_i \, x$. In this case, any rule is *myopic transfer-proof* by definition and therefore *transfer-proofness* trivially implies *myopic transfer-proofness*.

The following rule shows that if a desirable object exists, then *transfer-proofness* does not necessarily imply *myopic transfer-proofness*. Without loss of generality, let x^+ be a "desirable object" for agent 1, *i.e.*, $x^+ \, P_1 \, \emptyset$.

Example 8. ***Conditional Serial Dictatorship Rule*** $\varphi^{csd(x^+,E_1,\{1,2\})}$
A conditional serial dictatorship rule $\varphi^{csd(x^+,E_1,\{1,2\})}$ is defined as follows: Let $x^+ \in K$ be such that $x^+ \; P_1 \; \emptyset$, and φ^d, $\varphi^{d'}$ be serial dictatorship rules such that for φ^d, lower-indexed agents come first and for $\varphi^{d'}$, the order of agents 1 and 2 is switched and for the rest, lower-indexed agents come first. For all $E \in \mathcal{E}$ such that $x^+ \in E_1$, let $\varphi^{csd(x^+,E_1,\{1,2\})}(E) \equiv \varphi^{d'}(E)$. For all $E \in \mathcal{E}$ such that $x^+ \notin E_1$, let $\varphi^{csd(x^+,E_1,\{1,2\})}(E) \equiv \varphi^d(E)$. On $\mathcal{R}_a$, $\mathcal{R}_{sr}$, and $\mathcal{R}_s$, $\varphi^{csd(x^+,E_1,\{1,2\})}$ is *efficient* and *transfer-proof*, but not *myopic transfer-proof* (agent 2 may benefit from transferring object x^+ to agent 1). $\diamond$

The second case when *transfer-proofness* and *myopic transfer-proofness* are not logically independent occurs when all objects are desirable, *i.e.*, for all $x \in K$ and all $i \in N$, $x \; P_i \; \emptyset$. In this case, all transfers are weakly endowment improving for the recipient and therefore *myopic transfer-proofness* implies *transfer-proofness*.

The following rule shows that if an undesirable object exists, then *myopic transfer-proofness* does not necessarily imply *transfer-proofness*. Without loss of generality, let x^- be an "undesirable object" for agent 1, *i.e.*, $\emptyset \; P_1 \; x^-$.

Example 9. ***Conditional Serial Dictatorship Rule*** $\varphi^{csd(x^-,E_1,\{1,2\})}$
A conditional serial dictatorship rule $\varphi^{csd(x^-,E_1,\{1,2\})}$ is defined as follows: Let $x^- \in K$ be such that $\emptyset \; P_1 \; x^-$, and φ^d, $\varphi^{d'}$ be serial dictatorship rules such that for φ^d, lower-indexed agents come first, and for $\varphi^{d'}$, starting with agent 3 lower-indexed agents come first and then agents 1 and 2 come at the end of this order. For all $E \in \mathcal{E}$ such that $x^- \in E_1$, let $\varphi^{csd(x^-,E_1,\{1,2\})}(E) \equiv \varphi^d(E)$. For all $E \in \mathcal{E}$ such that $x^- \notin E_1$, let $\varphi^{csd(x^-,E_1,\{1,2\})}(E) \equiv \varphi^{d'}(E)$. Let $n \geq 3$. Then, for any separable preference profile, $\varphi^{csd(x^-,E_1,\{1,2\})}$ is *efficient* and *myopic transfer-proof*. However, even for profiles with additive preferences, $\varphi^{csd(x^-,E_1,\{1,2\})}$ may not be *transfer-proof* (agents 1 and 2 both may benefit if agent 2 transfers object x^- to agent 1). If $n = 2$, then for any separable preference profile, $\varphi^{csd(x^-,E_1,\{1,2\})}$ is *transfer-proof* as well. $\diamond$

All our results remain valid if we use *myopic transfer-proofness* instead of *(farsighted) transfer-proofness*. Even though *efficiency* does not imply *myopic transfer-proofness* when $n = 2$, we can still establish the compatibility of *myopic transfer-proofness* with *efficiency* and *individual rationality* for two agents. In fact, restricted serial dictatorship rules (defined in Section 3.2 on the domain of separable preferences in Example 5 and Footnote 5) are *myopic transfer-proof*.

We extend the definition of restricted (serial) dictatorship rules to the domain of unrestricted preferences $\mathcal{R}_u$. Let $N = \{1, 2\}$, $R \in \mathcal{R}_u^N$, and $E \in \mathcal{E}$. For all $j \in N$, let $\bar{E}_j$ be the most preferred subset of E_j for agent j, that is, for all $S \subseteq E_j$, $\bar{E}_j \; R_j \; S$.

Example 10. ***Restricted (Serial) Dictatorship Rule*** $\varphi^{rd(i)}$
Let $N = \{1, 2\}$ and $i \in N$. For all $R \in \mathcal{R}_u^N$ and all $E \in \mathcal{E}$, $\varphi^{rd(i)}$ picks the unique best allocation for agent i in $\mathcal{PI}(E)$ that is *individually rational* for agent $j \neq i$ with respect to $\bar{E}_j$, that is, $\varphi_j^{rd(i)}(E) \; R_j \; \bar{E}_j$. By construction, $\varphi^{rd(i)}$ is *efficient* and *individually rational*. $\diamond$

Next, we show that $\varphi^{rd(i)}$ is *myopic transfer-proof*.

Proposition 2. *For two agents with unrestricted preferences, restricted dictatorship rules are myopic transfer-proof.*

Proposition 2 remains valid on $\mathcal{R}_a$, $\mathcal{R}_{sr}$, $\mathcal{R}_s$, and $\mathcal{R}_u$.

Proof: Let $N = \{1, 2\}$, $\varphi = \varphi^{rd(1)}$, $R \in \mathcal{R}_u^N$, and $E \in \mathcal{E}$. We prove that neither agent can benefit from transferring one of her objects to the other agent. The proof that neither agent can benefit from transferring several objects follows by applying the "one-object-argument" for each object and invoking transitivity of preferences.

Case 1: Agent 1 transfers $x \in E_1$ to agent 2. Let $E_2' \equiv (E_2 \cup x)\, R_2\, E_2$. Let $A \equiv \varphi(E)$ and $B \equiv \varphi(E_1 \backslash x, E_2')$. Suppose $B_1\, P_1\, A_1$. Since $\bar{E} \in \mathcal{I}(E)$, by the definition of φ, $A_1\, R_1\, \bar{E}_1$. By *individual rationality*, $B_2\, R_2\, \bar{E}_2'\, R_2\, E_2'$. Note also that $\bar{E}_2'\, R_2\, \bar{E}_2$. Then, by transitivity, $B_1\, P_1\, \bar{E}_1$ and $B_2\, R_2\, \bar{E}_2$. Hence, there exists $C \in \mathcal{PI}(E)$ such that $C_1\, R_1\, B_1$ and $C_2\, R_2\, B_2$. Thus, $C_1 P_1 A_1$ and $C_2 R_2 \bar{E}_2$, which contradicts the assumption that A is the best allocation for agent 1 in $\mathcal{PI}(E)$.

Case 2: Agent 2 transfers $x \in E_2$ to agent 1. Let $(E_1 \cup x)\, R_1\, E_1$ and $E_2' \equiv E_2 \backslash x$. Let $A \equiv \varphi(E)$ and $B \equiv \varphi(E_1 \cup x, E_2')$. Suppose $B_2\, P_2\, A_2$. Then, by *efficiency*, $A_1\, P_1\, B_1$. By *individual rationality*, $B_1 R_1 (E_1 \cup x)$. By the definition of φ^r, $A_2 R_2 \bar{E}_2$. Note that $\bar{E}_2 R_2 \bar{E}_2'$. Then, by transitivity, $A_1\, P_1\, (E_1 \cup x)$ and $A_2\, R_2\, \bar{E}_2'$. Hence, there exists $C \in \mathcal{PI}(E_1 \cup x, E_2')$ such that $C_1\, R_1\, A_1$ and $C_2\, R_2\, A_2$. Thus, $C_1\, P_1\, B_1$ and $C_2\, R_2\, \bar{E}_2'$, which contradicts the assumption that B is the best allocation for agent 1 in $\mathcal{PI}(E_1 \cup x, E_2')$. □

The following example describes a class of rules that are all *efficient*, *individually rational*, and *myopic transfer-proof*.

Example 11. ***Restricted Conditional Dictatorship Rule*** $\varphi^{rcd(x,\tilde{E})}$
Let $N = \{1, 2\}$ and $x \in K$. For all $R \in \mathcal{R}_{sr}^N$ and all $E \in \mathcal{E}$ such that $x \in \bigcup_{i \in N} \tilde{E}_i$, $\varphi^{rcd(x,\tilde{E})}(E) = \varphi^{rd(1)}(E)$. For all $R \in \mathcal{R}_{sr}^N$ and all $E \in \mathcal{E}$ such that $x \notin \bigcup_{i \in N} \tilde{E}_i$, $\varphi^{rcd(x,\tilde{E})}(E) = \varphi^{rd(2)}(E)$. Then, $\varphi^{rcd(x,\tilde{E})}$ is *efficient*, *individually rational*, and *myopic transfer-proof*. ◇

Many other restricted conditional dictatorship rules that are *myopic transfer-proof* and are similar to those in Example 11 can be constructed. For instance, one can condition the choice of the restricted dictator differently on the set of collectively owned objects, *e.g.*, by $K' \cap \bigcup_{i \in N} \tilde{E}_i \neq \emptyset$ instead of $x \in \bigcup_{i \in N} \tilde{E}_i$. Hence, the class of rules that are *efficient*, *individually rational*, and *myopic transfer-proof* for two agents with separable and responsive preferences is very large.

The next example demonstrates that for more than two agents, a restricted serial dictatorship rule may be manipulable by transfers. This result holds for any subdomain of $\mathcal{R}_u$ that includes the domain of additive preferences $\mathcal{R}_a$, in particular, for $\mathcal{R}_a$, $\mathcal{R}_{sr}$, $\mathcal{R}_s$, and $\mathcal{R}_u$.

Example 12. Let $N = \{1, 2, 3\}$, $E = (E_1, E_2, E_3)$ be such that $E_1 = a$, $E_2 = bc$, $E_3 = de$, and $(R_1, R_2, R_3) \in \mathcal{R}_a^N$ be the same as in Example 7.

If agent 1 is the restricted dictator, then the restricted serial dictatorship rule picks (cd, ae, b). However, if agent 3 transfers object e to agent 2, for the resulting exchange market the restricted serial dictatorship rule picks (bce, d, a). Hence, agent 3 consumes a, which she prefers to b, in violation of *myopic transfer-proofness*. Since agent 2 receives d, which she prefers to ae, we also have a contradiction to *weak transfer-proofness*. ◇

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